Department of Electronics and Telecommunication Engineering

University of Moratuwa
Sri Lanka

Semester I Examination 2005 - B.Sc. Engineering Level 4

EN 407 Robotics

Answer all questions Time allowed: **Two hours**

[Q1]. {A} and {B} are two coincident coordinate frames. While {A} stays still, {B} rotates about x, z, and y fixed axes of {A} by angles α, γ, and β in that respect. Frame {B} also undergoes a translation that brings its origin to the position ${}^{A}P_{Bo}$ in frame {A}.

(a) Derive the expression for the rotation matrix ${}^{A}_{B}R$ in terms of basic rotation matrices.

(b) Construct the homogeneous transformation matrix ${}^{A}_{B}T$ in terms of ${}^{A}_{B}R$ and ${}^{A}P_{Bo}$

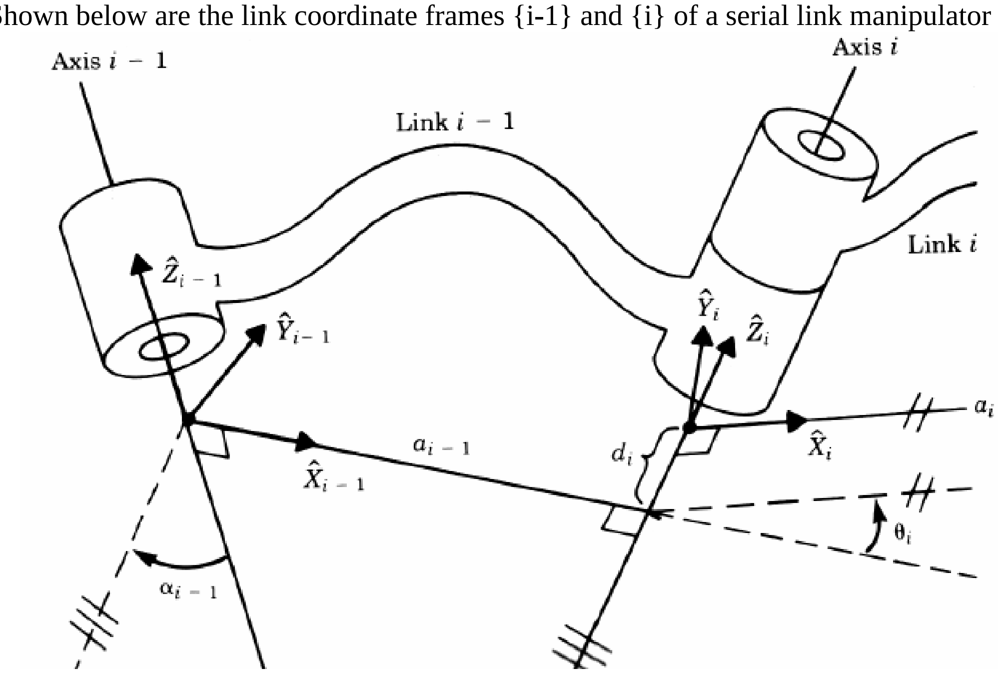
(c) Sketch the two coordinate frames and derive an expression for ${}^{B}_{A}T$

(d) Calculate ${}^{A}_{B}T$ when $\alpha = 60^0, \gamma = -30^0, \beta = 90^0$, and ${}^{A}P_{Bo} = [1,1,-2]^T$ and determine ${}^{A}Q$ when ${}^{B}Q = [1,0,2]^T$

Note: The basic rotation matrices are:

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix} \quad R_Y(\theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \quad R_Z(\theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[Q2]. Shown below are the link coordinate frames {i-1} and {i} of a serial link manipulator



(a) Construct an expression for the link transformation matrix ${}^{i-1}_{i}T$ in terms of basic rotation matrices (R′s) and basic translation matrices (D′s).

(b) The DH link-joint parameters of the first three links of PUMA560 robot manipulator are given in the table below. Write expressions for ${}^{0}_{1}T$, ${}^{1}_{2}T$, and ${}^{2}_{3}T$ in terms of basic rotation matrices and basic translation matrices (**Note**: specify respective angles, axes of rotation, translations and direction of translations. However, it is not required to compute matrix elements or to multiply matrices).

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	0	θ_2
3	0	431.8	149.1	θ_3

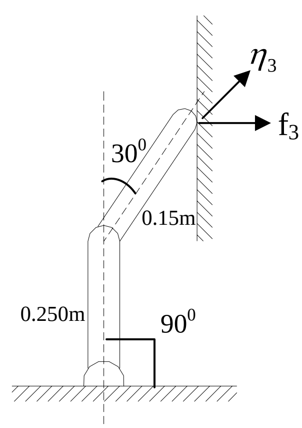
(c) Using the template for ${}^{i-1}_{i}T$, calculate the homogeneous transformation matrices ${}^{0}_{1}T$, ${}^{1}_{2}T$, ${}^{2}_{3}T$, and ${}^{0}_{3}T$ when PUMA 560 robot is at $[90^0,0^0,60^0]$ arm configuration.

Note: The template for ${}^{i-1}_{i}T$ is

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(d) Determine the origin of frame {3} with respect to frame {1}.

[Q3]. The manipulator shown below is in static equilibrium while exerting a force f_3 and a moment η_3 against a rigid wall.



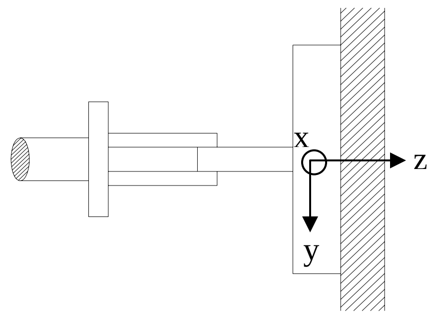
(a) Sketch the manipulator and draw the link coordinate frames {0},{1},{2}, and {3}
[**Hint**: Locate {3} at the extreme end (point of contact) of the robot manipulator]

(a) Derive expressions for f_2, η_2, f_1, and η_1 in terms of f_3, η_3

[**Note**: the rotation matrices ${}^1_2R, {}^2_3R$ and position vectors ${}^1P_2$ ${}^2P_3$ are known quantities].

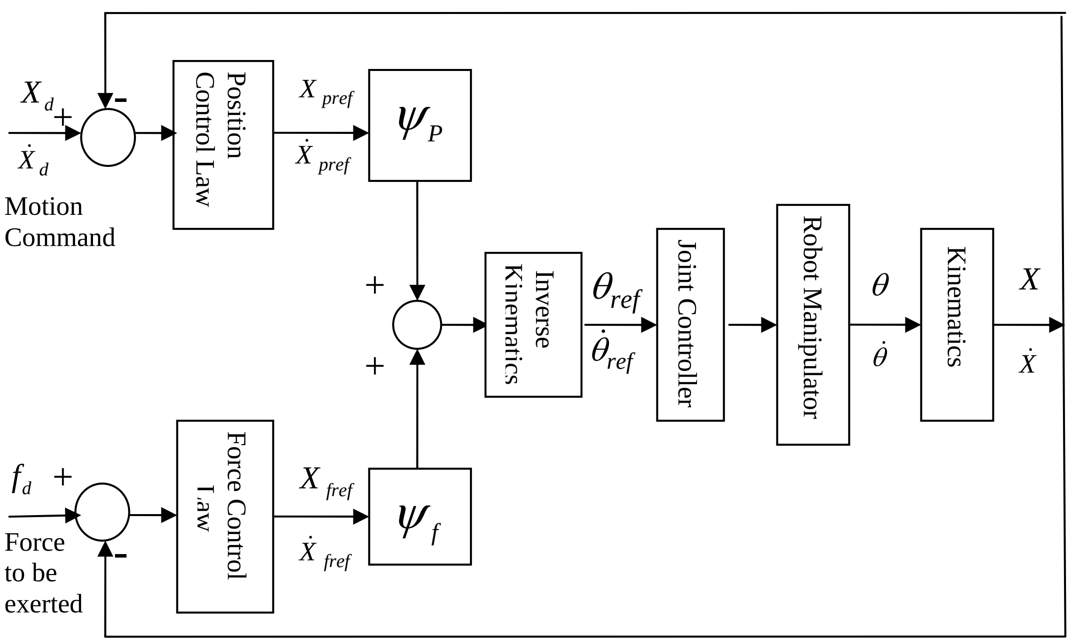
(b) The manipulator has to exert a force of 10N normal to the wall. Calculate f_2, η_2, η_1, and then calculate the required torques τ_1 and τ_2 at the two joints in order to maintain static equilibrium.

[Q4]. Figure below shows a robot manipulator used for window washing. The manipulator has to apply and maintain a constant predetermined force normal to the window while moving the washing sponge over the entire window surface.



(a) Identify the force/position constraints in X,Y,Z directions.

(b) Shown below is a hybrid force/position robot controller for the above application. Construct the selection matrices ψ_p and ψ_f.



(c) The end-effector compliant motion is governed by the relationship $F = K\delta x$, where F is the force to be generated against δx, the end-effector deflection in 3-space, and K is the end-effector stiffness. Starting from this relationship, derive the expression for corresponding joint torques $\tau = J^T(\theta)KJ(\theta)\delta\theta$ required to implement the compliant motion of the end-effector.

(d) Describe "active compliance" and "passive compliance" in robot force control